Optimal Design of a Stewart-Gough Platform for Multi-Directional Additive Manufacturing

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***Abstract* - This paper presents a conceptual design of a novel platform to carry out multi-directional Fused Deposition Modelling (FDM) process. Conventional Additive Manufacturing (AM) technologies are layer based with linear motions in three Cartesian axes. These techniques have limitations in the form of poor part accuracy at angles oblique to the build direction, slower build speed and additional support structure requirement. The limitation of build direction results in poor surface finish due to aliasing (or layer stair-stepping) and adverse material properties in certain directions. Various solutions were developed in the past to address these issues that resulted into increased build time and adversely affected the machine autonomy. These drawbacks limit the capabilities of the AM process with respect to other manufacturing processes. The present study proposes the use of multidirectional AM to address some of these issues by allowing 6-axis motion between tool and base for FDM process. To enable this, a number of robotic architectures are discussed in the paper and a qualitative criterion is evolved to compare them for the use in FDM. One such popular architecture namely, the Stewart-Gough platform (SGP) is explored in detail and its capability as a viable platform for FDM is illustrated. The design of SGP for multidirectional FDM is realized by formulating it as an optimization problem. The details of the optimization formulation and the consequent results are discussed at length. The proposed design of the multidirectional AM will not only minimize post-processing requirements but also allow for faster build-up of the part. Additionally, it is expected to offer advantages such as building around inserts, simplified slicing mechanism and control of isotropic material properties.**

***Keywords*— Multi-Directional Additive Manufacturing, Stewart-Gough Platform, Fused Deposition Modelling, Optimization**

1. INTRODUCTION

Additive or layered manufacturing techniques originated as Rapid Prototyping (RP) solution in variety of industries to create a system or part for representation before commercialization. Subsequent improvements in the quality of parts from these machines resulted in realization of these techniques as an alternative to produce finished components [1]. AM processes fabricate 3D components directly from CAD models by slicing it into 2D layers followed by controlled accumulation of material in each layer. Such an approach of fabricating the components has advantages such as, simplified tool path planning and hardware design as well as the capability to manufacture complex shapes with relative ease. However, the process suffers from few drawbacks, which limit its applications compared to other manufacturing processes. Some of the major drawbacks are:

- *Stair-casing (aliasing) effect*: Surfaces with non-parallel orientation to the build plate exhibit jagged edges reducing the accuracy and surface finish of the built component. It is primarily due to approximation of 3D geometry with layers of uniform thickness [2] as demonstrated in Fig. 1

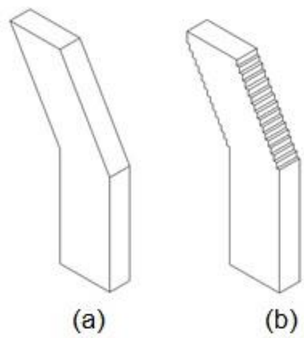

Fig 1. Stair-casing Effect on Build of Part (a) CAD model of actual part and the (b) Component fabricated using AM

- *Non-uniform material properties*: The build orientation has major impact on properties of the component. The decision related to optimal part orientation for uniform material properties is challenging and an unsolved process-planning problem [2]. As the component is built along single axis, properties normal to the build plate are only controlled which may be undesirable from a functional viewpoint.
- *Limited tool motions*: The tool motions are limited in AM system that do not allow building of material around inserts, e.g., embedded electric or optical systems.

- *Support structure requirements*: If component requires building along oblique direction, it necessitates use of support structures. The use of support structure increases build time and cost as well as post-process removal operations.

Few solutions have been suggested in the past literature to address these problems. To improve surface finish, controlled cure depth [3-4], post-processing techniques [5-6] and meniscus methods [7-8] have been developed. Techniques such as model shape modification [9] and hybrid process development [10] have been employed to enhance the fabrication capability and building-around-inserts. Song et al. [11] observed that these approaches improve one or a few drawbacks only. This is primarily due to use of single build direction and uniform layer thickness in fabrication of the component. It was highlighted that building along multiple directions using non-uniform layer thickness can solve these problems largely. The study examined feasibility of achieving multi-axis motion between extruder head and base plate using Stewart mechanism. It was demonstrated that the proposed mechanism helps in collectively addressing previously listed problems. This study focussed on development of Stewart mechanism as a concept and examining the feasibility of the same in AM applications. The FDM process is chosen as AM variant chosen in the present study for consistency with Song et al. [11]. The proposed design was not optimized for given workspace dimensions. The present paper is an attempt to derive an optimal design of Stewart-gouge platform for FDM process. The proposed design offers larger workspace and better dexterity compared to SGP configuration proposed by Song et al. [11].

Henceforth, the paper is organized as follows. Section 2 discusses different robot architectures available for multi axis motion and highlights strengths of the SGP. Section 3 discusses the kinematic model of the SGP along with associated terminology. It also summarizes design requirements in terms of build volume and other kinematic parameters. Section 4 presents formulation of kinematic optimization problem in the form of identifying decision variables, constraints, optimization algorithm, determination of optimal design parameters and further work. Section 5 summarizes finding of the present work.

2. ROBOTIC ARCHITECTURES FOR MULTI-DIRECTIONAL AM

Primarily, two types of robotic architectures are used for manufacturing applications involving multi-axis motions; Serial and Parallel. This section presents comparison of robotic architectures and their suitability to AM. The section also discusses other popular hybrid robotic architectures in the context of suitability for AM. The comparison of architecture is done based on four important criteria;

- Workspace to Footprint ratio (W/F): Workspace of the robot refers to the total reachable volume of its end effector in the space and the nominal length of its base determines its footprint. Higher workspace to footprint ratio of a robotic configuration ensures compactness of the design and enables portability. In general, serial configurations have higher W/F ratio than the parallel configuration.
- Nature of Workspace: Serial robots have simple and regular workspace. The workspace can be segmented into independent Cartesian and orientation spaces. Therefore, the problem of optimizing the workspace, tool path planning and control is fairly simplified. The workspace for parallel configuration is highly irregular and it cannot be segmented.
- Accuracy and error propagation: The primary disadvantage of serial configuration is lower accuracy compared to parallel configuration. An error in individual joint of a serial configuration propagates to other joints and finally in the workspace. This necessitates modelling of additional feedback and error compensation system for enhancement of accuracy. The closed loop structure of parallel configuration ensures that joint errors do not propagate.
- Rigidity: Most serial robots lack rigidity of links thus making tool control difficult. Some of the modern industrial configuration like ABB uses reinforced joints for high rigidity but it is quite expensive. The parallel configurations have inherently higher rigidity owing to closed loop structure.

Serial robotic configurations are not explored for manufacturing applications due to their high cost and low rigidity along with requirement of complex feedback systems. Apart from these two configurations, hybrid structure, e.g., delta robot was also proposed in the literature, which combines advantages of both architectures. Figure 2 shows hybrid delta manipulator developed by Fiore et al. [12] with an agile eye on the platform for orientation. The configuration was developed for injection moulding applications therefore, the extruder is kept stationary and motion is given to the platform. This configuration adversely affects build time and imposes difficulty in controlling material flow. There has been considerable interest in using parallel kinematic machines such as SGP for manufacturing applications in recent years. The SGP has been successfully implemented for machining processes requiring multi-axis motions in limited workspace [13]. The present work proposes SGP as an alternate configuration for AM application. Song et al. [11] proposed working prototype of SGP for workspace with size of 100mm x 100mm x 100mm with pitch and roll motions in the range of -30° to +30°. This paper proposes an improved design of similar SGP with optimal dexterity and larger workspace.

Fig 2. Delta Manipulator for Metal Injection Moulding [5]

3. KINEMATIC MODEL OF STEWART-GOUGH PLATFORM

The schematic diagram of SGP is shown in Fig. 3, which consists of six active prismatic pairs connecting the fixed base and moving platform on which the extruder is to be mounted. The linear actuators are connected to the top and bottom using passive universal and spherical joints, which transmit motion to the moving platform. The workspace requirements considered for the design of platform are as follows:

- The extruder should be able to translate with a cube of 110mm x 110mm x 110mm.
- It should have roll and pitch motions in the range of +45° to -45° within the cube.

The objective of kinematic modeling is to establish the relationship between joint space and task space. The task space is given by coordinates $(x, y, z, \alpha, \beta, \gamma)$ and the joint space is considered using length of the struts $(l_1, l_2, l_3, l_4, l_5, l_6)$. It can be seen from Fig. 3 that the fixed frame of reference $\{B\}$ is attached to the base with its origin O_B at the center of the base circle. The moving frame of reference $\{P\}$ is attached to the platform with its origin O_P at the centre of the platform. The moving frame is also the tool frame without loss of generality. The passive joints connecting linear actuators with the base and platform are located at points B_i and P_i, where $i = 1,2,\ldots,6$. The position of the passive joints are described using the vectors $\boldsymbol{b}_i$ and $\boldsymbol{p}_i$.

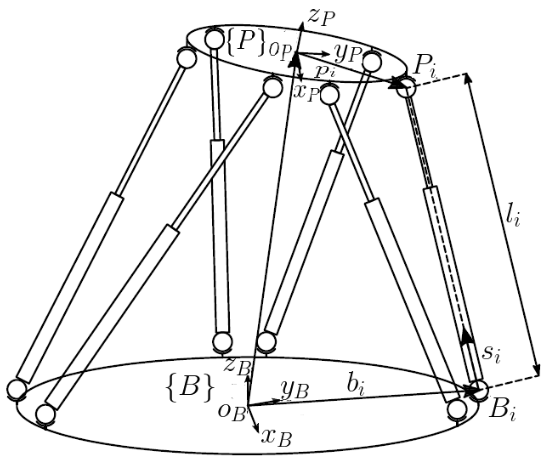

Fig 3. Schematic Diagram of a SGP

The leg lengths are denoted as l_i whereas the direction is along unit-vector $\boldsymbol{s_i}$. The position and orientation of the tool is described using position vector ${}^B\boldsymbol{t}$ and the rotation matrix ${}^B\boldsymbol{R}_p$ respectively. Here, the leading superscript implies the frame in which the vector is expressed. The rotation matrix ${}^B\boldsymbol{R}_p$ is used to express the orientation of frame $\{P\}$ with respect to frame $\{B\}$. Among different ways of representing rotation matrix, Roll-Pitch-Yaw (α, β, γ) presentation is adapted in this study. Equation 1 describes closed-loop equation for the i^{th} limb of the SGP.

$$l_i\ {}^B\boldsymbol{s}_i = {}^B\boldsymbol{t} + {}^B\boldsymbol{R}_p\ {}^P\boldsymbol{p}_i - {}^B\boldsymbol{b}_i \qquad (1)$$

The Euclidean norm of Eq. (1) determines leg length l_i. Differentiating Eq. (1) with respect to time and rearrangement of terms gives the Jacobian of the system which transforms joint velocities into platform velocities. The Jacobian in Eq. (2) forms the basis in deriving optimal design of the SGP.

$$\dot{\boldsymbol{l}} = \boldsymbol{J}\dot{\boldsymbol{q}} \qquad (2)$$

Here $\dot{\boldsymbol{l}}$ is actuator velocity vector, $\boldsymbol{J}$ is the Jacobian and $\dot{\boldsymbol{q}}$ is velocity vector of the tool point. Re-writing equation in the matrix-vector multiplication form yields the 6 x 6 Jacobian transformation matrix $\boldsymbol{J}$.

$$\dot{\boldsymbol{l}} = \begin{bmatrix} \dot{l_1} \\ \dot{l_2} \\ \dot{l_3} \\ \dot{l_4} \\ \dot{l_5} \\ \dot{l_6} \end{bmatrix} \boldsymbol{J} = \begin{bmatrix} \boldsymbol{s}_1^T & (\boldsymbol{Ra}_1 \times \boldsymbol{s}_1)^T \\ \boldsymbol{s}_2^T & (\boldsymbol{Ra}_2 \times \boldsymbol{s}_2)^T \\ \boldsymbol{s}_3^T & (\boldsymbol{Ra}_3 \times \boldsymbol{s}_3)^T \\ \boldsymbol{s}_4^T & (\boldsymbol{Ra}_4 \times \boldsymbol{s}_4)^T \\ \boldsymbol{s}_5^T & (\boldsymbol{Ra}_5 \times \boldsymbol{s}_5)^T \\ \boldsymbol{s}_6^T & (\boldsymbol{Ra}_6 \times \boldsymbol{s}_6)^T \end{bmatrix} \dot{\boldsymbol{q}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} \qquad (3)$$

4. KINEMATIC OPTIMIZATION

There are few research attempts related to optimal design of SGP based on the Jacobian which can be found from [14], [15], [18]. The Jacobian in Eq. 3 provides quantitative information about force and motion transmission capabilities of a parallel manipulator from the joint to task space. Molina et al. [14] discuss Jacobian based indices for optimal design of a Semi-Regular Stewart-Gough Platform (SRSGP). From AM viewpoint, Global Conditioning Index (GCI) will be of primary interest while designing the platform [15]. The reciprocal of condition number (ҁ) of the Jacobian is the measure of manipulator dexterity at a given pose.

$$0 \leq \frac{1}{ҁ} \leq 1 \tag{4}$$

The value of 0 for condition number implies singular position, i.e., the manipulator loses one or more degrees of freedom and any change in the joint variables does not affect the platform. The value of 1 for condition number indicates isotropy, i.e., the platform is able to transmit motion uniformly in all the directions. As the Jacobian is a function of the current pose of the platform, it is measure of the local dexterity of the platform. The global measure of dexterity, the GCI is defined using Eq. (5).

$$GCI = \frac{\int_w \frac{1}{ҁ} dw}{\int_w dw} \tag{5}$$

The greater value of GCI implies larger dexterity of SRSGP configuration over entire workspace. Therefore, GCI should be maximized while deriving optimal configuration. For discrete workspace, the GCI can be formulated as Eq. (6).

$$GCI = \frac{1}{n}\sum_{i=1}^{n} \frac{1}{ҁ} \tag{6}$$

The manipulator design problem is formulated as a nonlinear constrained optimization problem in the present study and genetic algorithm is employed to solve the optimization problem. Subsequent sub-sections describe formulation of optimization problem and solution of the same using Genetic Algorithm (GA).

A. Design Variables and Constraints

Molina et al. [14] described an SRSGP completely using five distinct design variables. Figure 4 shows these design variables along with constraints. The design variables along with bounds are listed below;

- r_b, radius of the base circle
  Maximum floor space of the platform should not exceed 1.5m x 1.5m therefore; radius of base circle is restricted at 0.75m

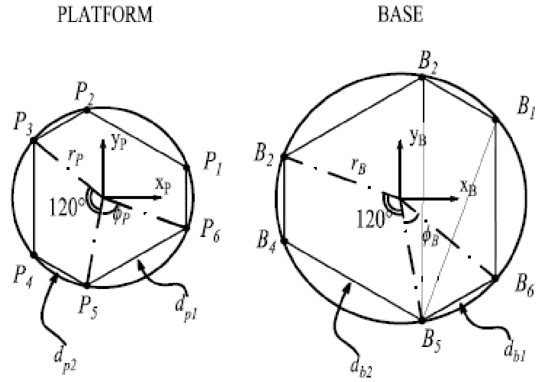

Fig 4. The SRSGP with Design Variables and Constraints

- r_p, radius of the platform circle
  The minimum value of platform circle radius is constrained by the size of the extruder unit that is to be mounted on it. The upper limit of platform radius is selected as 0.15m in the present study.
- φ_b , spacing angle between a set of passive joints at the base ($B_1 - B_2$, $B_3 - B_4$, $B_5 - B_6$). An upper limit of 120° is defined to prevent leg crossover.
  φ_p, spacing angle between a set of passive joints at the base($P_1 - P_2$, $P_3 - P_4$, $P_5 - P_6$). An upper limit of 120° is defined to prevent leg crossover in this case too.
- h , the distance between origin of the base and platform frames when it is in neutral position. The neutral position is defined when the base and the platform centres are parallel and coincident. The height of the platform is constrained by components of the system enclosed by the structure.

The design space can be represented as a vector in Eq. (7).

$$\boldsymbol{x} = [r_b \quad r_p \quad \varphi_b \quad \varphi_p \quad h] \tag{7}$$

Table 1 summarizes limits of design variables based on discussion above.

TABLE 1 Design Variables and Bounds

$r_b(m)$	$r_p(m)$	$\varphi_b(rad)$	$\varphi_p(rad)$	$h(m)$
0.25-0.75	0-0.15	0-$2\pi/3$	0-$2\pi/3$	0.2-0.5

The kinematic model can be rewritten using Eq. (8) and (9).

$$\boldsymbol{b_i} = \begin{bmatrix} \boldsymbol{r_b}\mathbf{cos}(\boldsymbol{\varphi_{B,i}}) \\ \boldsymbol{r_b}\mathbf{sin}(\boldsymbol{\varphi_{B,i}}) \\ \mathbf{0} \end{bmatrix}, \boldsymbol{p_i} = \begin{bmatrix} \boldsymbol{r_p}\mathbf{cos}(\boldsymbol{\varphi_{P,i}}) \\ \boldsymbol{r_p}\mathbf{sin}(\boldsymbol{\varphi_{P,i}}) \\ \mathbf{0} \end{bmatrix} \quad (1)$$

Here,

$$\varphi_{B,i} = \frac{i\pi}{3} - \frac{\varphi_B}{2}\ , \varphi_{P,i} = \frac{i\pi}{3} - \frac{\varphi_P}{2} \quad i = 1,3,5$$
$$\varphi_{B,i} = \varphi_{B,i} + \varphi_B\ ,\ \varphi_{P,i} = \varphi_{P,i-1} + \varphi_P \quad i = 2,4,6 \tag{2}$$

B. Formulation of Constraints

The constraints are formulated considering the physical limitations of the design and avoidance of singularities. It is required that the limbs of the platform should not share the same passive joint. The construction of such a joint is extremely difficult and it limits the motion of platform due to collisions. As setting two spacing angles φ_B and φ_P between 0° and 120° does not account for the finite radius of the joints, it is necessary to ensure that two joints do not coincide. Therefore, an additional parameter, spacing distance d_{min} is defined. It is defined as the minimum distance between two passive joints. The symmetry of platform ensures that there exists only two such distances for the base, d_{b1} and d_{b2} as illustrated in Fig. 4. These distances can be defined using cosine rule as Eq. (10).

$$d_{b1} = \sqrt{2r_b^2 - 2r_b^2\cos(\varphi_B)}$$
$$d_{b2} = \sqrt{2r_B^2 - 2r_B^2\cos(120 - \varphi_B)} \tag{3}$$

Further, distances for the bottom platform can be defined in a similar manner using Eq. (11).

$$d_{p1} = \sqrt{2r_P^2 - 2r_P^2\cos(\varphi_P)}$$
$$d_{p2} = \sqrt{2r_P^2 - 2r_P^2\cos(120 - \varphi_P)} \tag{4}$$

In order to account for the size of passive joints, 50 mm distance is set for the base and 25 mm for the platform. These constraints are written as Eq. (12).

$$0.05 - d_{b1} \le 0 \qquad 0.05 - d_{b2} \le 0$$
$$0.025 - d_{p1} \le 0 \qquad 0.025 - d_{p2} \le 0 \tag{5}$$

The kinetostatic performance indices represent bulk values therefore; it cannot determine singularities encountered by the SRSGP in its workspace. To avoid singularities, the condition number should be above a certain value. This value is chosen to be 0.01 in the present study. This constraint can be written as Eq. (13).

$$0.01 - \min\left(\frac{1}{\kappa}\right) \le 0 \tag{6}$$

An additional constraint is required for actuators of certain minimum length and stroke. This can be specified using constraint Eq. (14) and (15).

$$l_{min} \le l_i \le l_{max} \tag{7}$$

Here

$$l_{max} - l_{min} = l_{stroke} \tag{8}$$

For the i^{th} leg of the SRSGP, the bounds on the height are used as a formality for the genetic algorithm. The height is indirectly constrained by leg length of the actuators which behave as active constraints on the system.

C. Workspace discretization

It is required to discretize the workspace for evaluating GCI numerically. The workspace has been represented as a finite set of poses $(x, y, z, \alpha, \beta, \gamma)$. The discretization can be achieved by two numbers; First, η_{ew} representing number of divisions in the Cartesian workspace and, Second, η_{cw} representing number of divisions in the Eulerian workspace. The number of poses generated is denoted by the letter N the value of which is determined by Eq.(16)

$$N = \eta_{cw}^3 \eta_{ew}^3 \tag{9}$$

If higher value of η_{ew} and η_{cw} is chosen in computation, the precision of calculated GCI will be better. But, it requires significantly larger computation time. The discretization values of $\eta_{cw} = 6$ and $\eta_{ew} = 5$ are chosen in the present optimization problem to ensure convergence and precision while maintaining fairly lesser number of poses to be computed.

D. Optimization

The optimization problem can be stated in the form of a minimization problem using Eq. (17).

$$minimize\ \ f(x) = -GCI \tag{10}$$

The objective function is subjected to non-linear constraints described in Eq. (11) and (12). The problem lacks an explicit analytical form. Standard gradient-based solvers cannot be used for such problems as it is not continuous and gradient is not well defined. Typically, evolutionary algorithms are used in literature [16] to solve such problems. In the present study, non-linear optimization problem has been solved using MATLAB Optimization Toolbox. Table 2 lists input parameters used in the GA based solution.

TABLE 2 GA Parameters

Parameters	Value
Population Size	50
Maximum Generations	100
Encoding Type	Double Vector
Selection Strategy	Stochastic Uniform
Mutation Type	Constraint Dependent
Crossover Type	Constraint Dependent

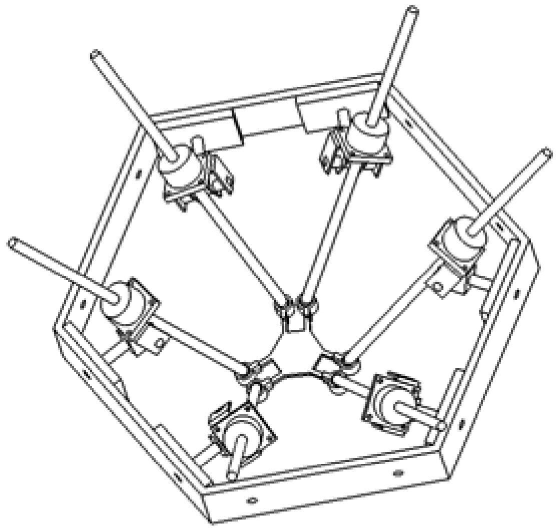

Fig 5. Schematic Diagram of SRSGP for AM

The GA converges after 30 generations and results are recorded. Table 3 summarizes output of GA after 30 generations. The convergence is achieved when the improvements in the fitness of the solution is less than a threshold value which is set as $1x10^{-6}$ in this case.

TABLE 3 Optimum Values Of Variables

GCI	r_b(m)	$r_p(m)$	φ_b(rad)	φ_p(rad)	$h(m)$
0.375	0.4	0.3	1.972	0.1	0.48

.

The dimension of each leg can be determined using inverse kinematics for the desired workspace. Based on output of GA, the following dimensions are determined for the linear actuator.

$$l_{min} = 0.26 \quad l_{max} = 0.9 \Rightarrow l_{stroke} = 640mm$$

A set of linear actuator with these dimensions can be used for building legs of the SRSGP. The future work focuses on building a prototype for AM applications using derived design dimensions along with the necessary control architecture. The conceptual model of SRSGP based AM solution is shown in Fig. (5).

5. CONCLUSIONS

The present study proposed systematic methodology to derive working dimensions of a novel SRSGP based solution for AM manufacturing application. The value of GCI for optimal design is within the acceptable range (0.4 - 0.7) recommended in the literature [14, 17]. Based on the value of GCI for optimal design, it can be inferred that the workspace is comparatively large for the size of the manipulator. Therefore, the proposed design of a manipulator can be treated as a good trade-off between workspace and dexterity.

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